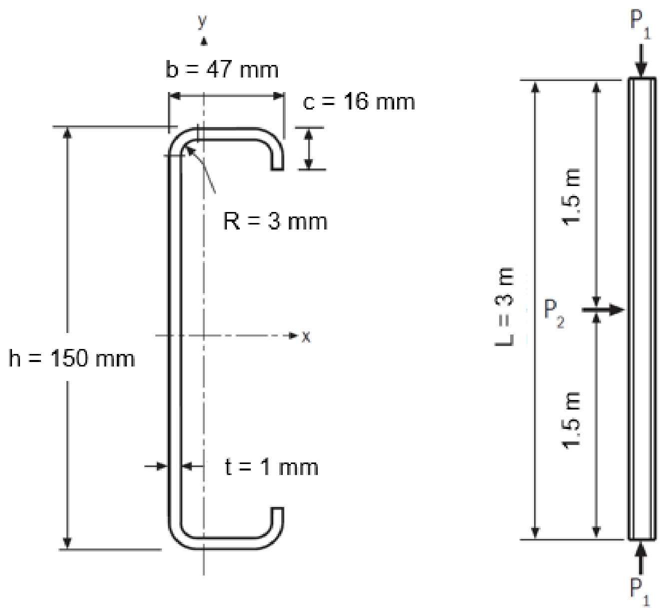
EC3 1-3 2006 CFFD Example 001

C-SECTION MEMBER WITH LIPS UNDER COMBINED COMPRESSION, BENDING, AND SHEAR

EXAMPLE DESCRIPTION

Compression, moment, and shear capacities and demand/capacity ratio are calculated for C section with lips at mid-height as shown below. It is simply supported with a length of 3.0 meters. The member is fully braced for flexural buckling about y-axis.

GEOMETRY, PROPERTIES AND LOADING



Dead: $P_1 = 500\ N, P_2 = 250\ N$
Live: $P_1 = 1000\ N, P_2 = 300\ N$

TECHNICAL FEATURES TESTED

- Axial compressive strength
- Major moment strength
- Shear strength
- Demand/Capacity ratio.

COMPUTER FILE: EC3 1-3 2006 CFFD EX001

Applicable Programs

- SAP2000

RESULTS COMPARISON

Independent results are hand calculated.

CONCLUSION

The results show exact match with independent results.

Benchmarks: SAP2000

Output Parameter	Program	Independent	Percent Difference
Axial - Flexural buckling $N_{b,Rd}$ (N)	39976	39961	0.04%
Axial – Torsional-Flexural buckling $N_{b,Rd}$ (N)	15071	15091	0.13%
Axial – Local & Distortional Buckling $N_{c,Rd}$ (N)	44255	44255	0.00%
Flexure – Lateral-Torsional Buckling $M_{b,Rd}$ $(N-mm)$	868257	868206	0.00%
Flexure – Local & Distortional Buckling $M_{c,Rd}$ $(N-mm)$	3097421	3097420	0.00%
Shear $V_{b,Rd}$ (N)	7888	7888	0.00%
D/C Ratio	0.971	0.971	0.00%

HAND CALCULATION

Properties:

Material: $E = 210{,}000\ N/mm^2,\ G = 80{,}770\ N/mm^2,\ f_{yb} = 350\ N/mm^2$

Section:

$h = 150\ mm,\ b = 47\ mm,\ t = 1\ mm,\ c = 16\ mm,\ r = 3\ mm$

$\rightarrow h_p = h - t = 150 - 1 = 149\ mm$

$\rightarrow b_p = b - 1 = 47 - 1 = 46\ mm$

$\rightarrow c_p = c - t/2 = 16 - 1/2 = 15.5\ mm$

Check for the effect of rounding of the corners:

$$\frac{r}{t} = \frac{3}{1} = 3 < 5 \rightarrow OK$$

$$\frac{r}{b_p} = \frac{3}{46} = 0.065 < 0.1 \rightarrow OK$$

Therefore, the effect of rounding of the corners can be neglected in calculation of section properties:

$A_g = 272\ (mm^2)$

$I_y = 925248.782\ (mm^4)$

$I_z = 84362.534\ (mm^4)$

$i_y = 58.324\ (mm)$

$i_z = 17.611\ (mm)$

$W_{el} = 12419.447\ (mm^3)$

$I_t = 90.667\ (mm^4)$

$I_w = 392274920\ (mm^6)$

$y_0 = 34.151\ (mm)$

$z_0 = 0.0\ (mm)$

Member: $K_y = K_T = 1.0$ for a pinned-pinned condition

$L_y = L_z = L_T = 3000\ mm$

Member is braced against flexural buckling about z-z axis.

$k_{yy} = k_{zz} = k_{zy} = k_{yz} = 1.0$

Loadings: Dead: $P_1 = 500\ N, P_2 = 250\ N$

Live: $P_1 = 1000\ N, P_2 = 300\ N$

Required strengths: for the section in the middle

$$N_{Ed} = 1.2D + 1.6L = 1.2 \times 500 + 1.6 \times 1000 = 2200\ (N)$$

$$M_{Ed} = 1.2D + 1.6L = 1.2 \times \frac{250 \times 3000}{4} + 1.6 \times \frac{300 \times 3000}{4} = 585000\ (N - mm)$$

$$V_{Ed} = 1.2D + 1.6L = 1.2 \times \frac{250}{2} + 1.6 \times \frac{300}{2} = 390\ (N)$$

Member Compression Capacity: the compression capacity is calculated considering the limit states of global buckling, and local and distortional buckling.

1. Local and distortional buckling:
The effective width method is utilized to calculate the nominal axial strength in consideration of local and distortional buckling with the compressive stress of $f_{yb} = 350\ (N/mm^2)$.

Check for the applicability of the method as the following conditions are satisfied:

$$\frac{b}{t} = \frac{47}{1} = 47 < 60 \rightarrow OK$$
$$\frac{c}{t} = \frac{16}{1} = 16 < 50 \rightarrow OK$$
$$\frac{h}{t} = \frac{150}{1} = 150 < 500 \rightarrow OK$$
$$\frac{c}{b} = \frac{16}{47} = 0.34 \rightarrow 0.2 < \frac{c}{b} < 0.6 \rightarrow OK$$

As the section is subjected to uniform compression and both flanges have identical dimension, they are considered as partially stiffened elements with a simple lip edge stiffener and have the same effective properties. The calculation below is only shown for the top flange:

$$\psi = 1$$
$$k_\sigma = 4$$
$$\varepsilon = \sqrt{\frac{235}{f_{yb}[N/mm^2]}} = \sqrt{\frac{235}{350}} = 0.8194$$
$$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{46/1}{28.4 \times 0.8194\sqrt{4}} = 0.988 > 0.673$$
$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{0.988 - 0.055(3+1)}{0.988^2} = 0.787 \leq 1.0$$
$$b_{eff} = \rho b_p = 0.787 \times 46 = 36.2\ (mm)$$
$$b_{e1} = b_{e2} = 0.5 b_{eff} = 0.5 \times 36.2 = 18.1\ (mm)$$

The lip is considered an outstand (unstiffened) element under uniform compression:

$$\frac{c_p}{b_p} = \frac{15.5}{46} = 0.337 < 0.35 \rightarrow k = 0.5$$
$$\bar{\lambda}_{p,c} = \frac{c_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{15.5/1}{28.4 \times 0.8194\sqrt{0.5}} = 0.942 > 0.748$$
$$\rho = \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{0.942 - 0.188}{0.942^2} = 0.85 \leq 1.0$$
$$c_{eff} = \rho c_p = 0.85 \times 15.5 = 13.17\ (mm)$$

The stiffener consisting of b_{e2} of the flange and c_{eff} of the lip (Figure 1) is subjected to distortional buckling (b_{e1} of the flange is not affected by distortional buckling and not included in the iterative procedure below):

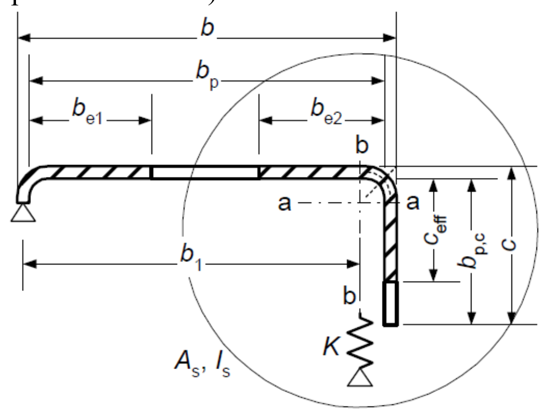


Figure 1 Edge Stiffener (Dubina et al., 2012)

<u>1st iteration:</u>

$$b_1 = b_2 = b_p - \frac{\frac{tb_{e2}^2}{2}}{t\left(b_{e2}+c_{eff}\right)} = 46 - \frac{1\times\frac{18.1^2}{2}}{1(18.1+13.17)} = 40.76\ (mm)$$

$$A_{s1} = A_{s2} = t\left(b_{e2}+c_{eff}\right) = 1(18.1+13.17) = 31.27\ (mm)$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1$$

$$K = \frac{Et^3}{4(1-\upsilon^2)}\frac{1}{\left(b_1^2h_p + b_1^3 + 0.5b_1b_2h_pk_f\right)}$$

$$= \frac{210{,}000\times 1^3}{4(1-0.3^2)}\frac{1}{(40.76^2\times 149 + 40.76^3 + 0.5\times 40.76\times 40.76\times 149\times 1)} = 0.131\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2b_{e2}^2 + 4b_{e2}c_{eff}^3 + t^2b_{e2}c_{eff} + c_{eff}^4\right)}{12\left(b_{e2}+c_{eff}\right)}$$

$$= \frac{1(1^2\times 18.1^2 + 4\times 18.1\times 13.17^3 + 1^2\times 18.1\times 13.17 + 13.17^4)}{12(18.1+13.17)} = 522.46(mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.131\times 210{,}000\times 522.46}}{31.27} = 242.88\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/242.88} = 1.2 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723\times 1.2 = 0.602$$

Since $\chi_d = 0.602 < 1.0 \rightarrow$ iteration is required.

<u>2nd iteration:</u>

b_{e2} of the flange and c_{eff} of the lip are subjected to reduced stress $\sigma_{com,Ed} = \chi_d f_{yb}/\gamma_{M0}$ such that:

$$\bar{\lambda}_{p,b,red} = \bar{\lambda}_{p,b}\sqrt{\chi_d} = 0.988 \times \sqrt{0.602} = 0.767 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{0.767 - 0.055(3+1)}{0.767^2} = 0.930 \le 1.0$$

$$b_{e2} = 0.5b_{eff} = 0.5\rho b_p = 0.5 \times 0.930 \times 46 = 21.39\ (mm)$$

$$\bar{\lambda}_{p,c,red} = \bar{\lambda}_{p,c}\sqrt{\chi_d} = 0.942 \times \sqrt{0.602} = 0.731 < 0.748 \rightarrow \rho = 1.0$$

$$c_{eff} = \rho c_p = 1.0 \times 15.5 = 15.5\ (mm)$$

$$b_1 = b_2 = b_p - \frac{\frac{tb_{e2}^2}{2}}{t\left(b_{e2} + c_{eff}\right)} = 46 - \frac{1 \times \frac{21.39^2}{2}}{1(21.39 + 15.5)} = 39.8\ (mm)$$

$$A_{s1} = A_{s2} = t\left(b_{e2} + c_{eff}\right) = 1(21.39 + 15.5) = 36.88\ (mm)$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1$$

$$K = \frac{Et^3}{4(1-\upsilon^2)}\frac{1}{\left(b_1^2 h_p + b_1^3 + 0.5b_1 b_2 h_p k_f\right)}$$

$$= \frac{210{,}000 \times 1^3}{4(1-0.3^2)}\frac{1}{(39.8^2 \times 149 + 39.8^3 + 0.5 \times 39.8 \times 39.8 \times 149 \times 1)} = 0.138\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e2}^2 + 4b_{e2}c_{eff}^3 + t^2 b_{e2}c_{eff} + c_{eff}^4\right)}{12\left(b_{e2} + c_{eff}\right)}$$

$$= \frac{1(1^2 \times 21.16^2 + 4 \times 21.16 \times 15.5^3 + 1^2 \times 21.16 \times 15.5 + 15.5^4)}{12(21.16 + 15.5)} = 815.9\ (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.138 \times 210{,}000 \times 815.9}}{36.88} = 269.7\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/269.7} = 1.139 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.139 = 0.646$$

Since $\chi_d = 0.646 \ne 0.602$ from previous iteration, more iterations are carried out and the final iteration gives:

$$\chi_d = 0.646.$$

$$b_{e2} = 20.93\ (mm)$$

$$c_{eff} = 15.39\ (mm)$$

$$b_{e1} = 18.1\ (mm)$$

The web is considered an internal (stiffened) element under uniform compression:

$$\psi = 1$$

$$k_\sigma = 4$$

$$\bar{\lambda}_{p,b} = \frac{h_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{149/1}{28.4 \times 0.8194\sqrt{4}} = 3.2 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{3.2 - 0.055(3+1)}{3.2^2} = 0.291 \le 1.0$$

$$h_{eff} = \rho h_p = 0.291 \times 149 = 43.344\ (mm)$$

$$A_{eff} = th_{eff} + 2tb_{e1} + 2\chi_d t\left(b_{e2} + c_{eff}\right)$$

$$= 1 \times 43.344 + 2 \times 1 \times 18.1 + 2 \times 0.646 \times 1(20.93 + 15.39) = 126.44\ (mm^2)$$

$$A_{eff} = 126.44\ (mm^2) < 266\ (mm^2) = A_g$$

$$\rightarrow N_{c,Rd} = \frac{A_{eff} f_{yb}}{\gamma_{M0}} = \frac{126.44 \times 350}{1.0} = 44255\ (N)$$

Because the section is symmetric about y-y axis, its effective properties are also symmetric about y-y axis, resulting in $e_{Ny} = 0 \rightarrow \Delta M_{y,Ed} = 0$

$$\bar{z} = \frac{\sum_i A_i z_i}{A} = \frac{2tb_p \frac{b_p}{2} + 2tc_p b_p}{A} = \frac{46 \times 46 + 2 \times 15.5 \times 46}{272} = 13.02\ (mm)$$

$$\bar{z}_{eff} = \frac{\sum_i A_{eff,i} z_i}{A_{eff}} = \frac{2tb_{e1}\frac{b_{e1}}{2} + 2\chi_d t b_{e2}\left(b_p - \frac{b_{e2}}{2}\right) + 2\chi_d t c_{eff} b_p}{A_{eff}}$$

$$= \frac{18.1 \times 18.1 + 2 \times 0.646 \times 20.93\left(46 - \frac{20.93}{2}\right) + 2 \times 0.646 \times 15.39 \times 46}{126.44} = 17.42\ (mm)$$

$$e_{Nz} = \bar{z}_{eff} - \bar{z} = 17.42 - 13.02 = 4.4\ (mm)$$

$$\Delta M_{z,Ed} = N_{Ed} e_{Nz} = 2200 \times 4.4 = 9680\ (N - mm)$$

2. Global buckling: includes flexural buckling and torsional and flexural-torsional buckling
   i. Flexural buckling: since the member is fully braced against flexural buckling about z-axis. In the program, this bracing is modeled by assigning such a small value of effective length factor K_{2minor} for the element.

$$N_{cr,y} = \frac{\pi^2 E I_y}{\left(K_y L_y\right)^2} = \frac{\pi^2 (210{,}000) 925248.782}{(1.0 \times 3000)^2} = 213076\ (N)$$

$$\bar{\lambda}_y = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,y}}} = \sqrt{\frac{126.44 \times 350}{213076}} = 0.456$$

For C section with lips, the buckling curve is b and $\alpha = 0.34$

$$\Phi_y = 0.5\left[1 + \alpha\left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^2\right] = 0.5[1 + 0.34(0.456 - 0.2) + 0.456^2] = 0.647$$

$$\chi_y = \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \bar{\lambda}_y^2}} = \frac{1}{0.647 + \sqrt{0.647^2 - 0.456^2}} = 0.903$$

$$N_{b,Rd} = \frac{\chi_y A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.903 \times 126.44 \times 350}{1.0} = 39961\ (N)$$

   ii. Torsional and flexural-torsional buckling:

$$i_0 = \sqrt{i_y^2 + i_z^2 + y_0^2 + z_0^2} = \sqrt{58.324^2 + 17.611^2 + 34.151^2 + 0^2} = 69.84\ (mm)$$

$$N_{cr,T}=\frac{1}{i_0^2}\left[GI_t+\frac{\pi^2 EI_w}{L_T^2}\right]=\frac{1}{69.84^2}\left[80{,}770\times 90.667+\frac{\pi^2 210{,}000\times 392274920}{(1.0\times 3000)^2}\right]=20022\ (N)$$

$$\beta=1-\frac{y_0^2+z_0^2}{i_0^2}=1-\frac{34.151^2+0^2}{69.84^2}=0.761$$

$$N_{cr,TF}=\frac{N_{cr,y}}{2\beta}\left[1+\frac{N_{cr,T}}{N_{cr,y}}-\sqrt{\left(1-\frac{N_{cr,T}}{N_{cr,y}}\right)^2+4\left(\frac{y_0}{i_0}\right)^2\frac{N_{cr,T}}{N_{cr,y}}}\right]$$

$$=\frac{213076}{2\times 0.761}\left[1+\frac{20022}{213076}-\sqrt{\left(1-\frac{20022}{213076}\right)^2+4\left(\frac{34.151}{69.84}\right)^2\frac{20022}{213076}}\right]=19548.2\ (N)$$

As $N_{cr,TF}=19548.2\ (N)<20022\ (N)=N_{cr,T}$

$$\rightarrow \bar{\lambda}_T=\sqrt{\frac{A_{eff}f_{yb}}{N_{cr,TF}}}=\sqrt{\frac{126.44\times 350}{19548.2}}=1.505$$

For C section with lips, the buckling curve for torsional-flexural buckling is b and $\alpha=0.34$

$$\Phi_T=0.5\left[1+\alpha\left(\bar{\lambda}_T-0.2\right)+\bar{\lambda}_T^2\right]=0.5[1+0.34(1.505-0.2)+1.505^2]=1.854$$

$$\chi_T=\frac{1}{\Phi_T+\sqrt{\Phi_T^2-\bar{\lambda}_T^2}}=\frac{1}{1.854+\sqrt{1.854^2-1.505^2}}=0.341$$

$$N_{b,Rd}=\frac{\chi_T A_{eff}f_{yb}}{\gamma_{M1}}=\frac{0.341\times 126.44\times 350}{1.0}=15090.6\ (N)$$

Member Flexural Capacity: the flexural capacity is calculated considering the limit states of lateral-torsional buckling, and local and distortional buckling.

1. Local and distortional buckling:
   The effective width method is utilized to calculate the nominal flexural strength in consideration of local and distortional buckling with the compressive stress in the top flange of $f_{yb}=350\ (N/mm^2)$. As the section is subjected to positive moment, the top flange is under compression and it is considered a partially stiffened element with a simple lip edge stiffener:

$$\psi=1$$

$$k_\sigma=4$$

$$\bar{\lambda}_{p,b}=\frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}}=\frac{46/1}{28.4\times 0.8194\sqrt{4}}=0.988>0.673$$

$$\rho=\frac{\bar{\lambda}_{p,b}-0.055(3+\psi)}{\bar{\lambda}_{p,b}^2}=\frac{0.988-0.055(3+1)}{0.988^2}=0.787\leq 1.0$$

$$b_{eff}=\rho b_p=0.787\times 46=36.2\ (mm)$$

$$b_{e1}=b_{e2}=0.5b_{eff}=0.5\times 36.2=18.1\ (mm)$$

The bottom flange is in tension and calculation of its effective width is not carried out.

The lip is considered an outstand (unstiffened) element under uniform compression:

$$\frac{c_p}{b_p} = \frac{15.5}{46} = 0.337 < 0.35 \rightarrow k = 0.5$$

$$\bar{\lambda}_{p,c} = \frac{c_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{15.5/1}{28.4 \times 0.8194\sqrt{0.5}} = 0.942 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{0.942 - 0.188}{0.942^2} = 0.85 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.85 \times 15.5 = 13.17\ (mm)$$

The stiffener consisting of b_{e2} of the top flange and c_{eff} of the top lip (Figure 1) is subjected to distortional buckling (b_{e1} of the top flange is not affected by distortional buckling and not included in the iterative procedure below):

1st iteration:

$$b_1 = b_p - \frac{\frac{tb_{e2}^2}{2}}{t\left(b_{e2} + c_{eff}\right)} = 46 - \frac{1 \times \frac{18.1^2}{2}}{1(18.1 + 13.17)} = 40.76\ (mm)$$

$$b_2 = 0$$

$A_{s1} = t\left(b_{e2} + c_{eff}\right) = 1(18.1 + 13.17) = 31.27\ (mm)$ (top flange)

$A_{s2} = 0$ as the bottom flange is in tension

$$k_f = \frac{A_{s2}}{A_{s1}} = 0$$

$$K = \frac{Et^3}{4(1 - \upsilon^2)} \frac{1}{\left(b_1^2 h_p + b_1^3 + 0.5 b_1 b_2 h_p k_{f1}\right)}$$

$$= \frac{210{,}000 \times 1^3}{4(1 - 0.3^2)} \frac{1}{(40.76^2 \times 149 + 40.76^3 + 0)} = 0.183\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e2}^2 + 4 b_{e2} c_{eff}^3 + t^2 b_{e2} c_{eff} + c_{eff}^4\right)}{12\left(b_{e2} + c_{eff}\right)}$$

$$= \frac{1(1^2 \times 18.1^2 + 4 \times 18.1 \times 13.17^3 + 1^2 \times 18.1 \times 13.17 + 13.17^4)}{12(18.1 + 13.17)} = 522.46 (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.183 \times 210{,}000 \times 522.46}}{31.27} = 286.62\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/286.62} = 1.105 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.105 = 0.671$$

Since $\chi_d = 0.671 < 1.0 \rightarrow$ iteration is required.

2nd iteration:

b_{e2} of the flange and c_{eff} of the lip are subjected to reduced stress $\sigma_{com,Ed} = \chi_d f_{yb}/\gamma_{M0}$ such that:

$$\bar{\lambda}_{p,b,red} = \bar{\lambda}_{p,b}\sqrt{\chi_d} = 0.988 \times \sqrt{0.671} = 0.81 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{0.81 - 0.055(3+1)}{0.81^2} = 0.9 \leq 1.0$$

$$b_{e2} = 0.5b_{eff} = 0.5\rho b_p = 0.5 \times 0.9 \times 46 = 20.7\ (mm)$$

$$\bar{\lambda}_{p,c,red} = \bar{\lambda}_{p,c}\sqrt{\chi_d} = 0.942 \times \sqrt{0.671} = 0.772 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{0.772 - 0.188}{0.772^2} = 0.98 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.98 \times 15.5 = 15.2\ (mm)$$

$$b_1 = b_p - \frac{\frac{tb_{e2}^2}{2}}{t\left(b_{e2} + c_{eff}\right)} = 46 - \frac{1 \times \frac{20.7^2}{2}}{1(20.7 + 15.2)} = 40.03\ (mm)$$

$$A_{s1} = t\left(b_{e2} + c_{eff}\right) = 1(20.7 + 15.2) = 35.9\ (mm)$$

$A_{s2} = 0$ as the bottom flange is in tension

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 0$$

$$K = \frac{Et^3}{4(1-v^2)}\frac{1}{\left(b_1^2 h_p + b_1^3 + 0.5b_1 b_2 h_p k_{f1}\right)}$$

$$= \frac{210{,}000 \times 1^3}{4(1-0.3^2)}\frac{1}{(40.03^2 \times 149 + 40.03^3 + 0)} = 0.19\ (N/mm^2)$$

$$I_s = \frac{t\left(t^2 b_{e2}^2 + 4b_{e2}c_{eff}^3 + t^2 b_{e2}c_{eff} + c_{eff}^4\right)}{12\left(b_{e2} + c_{eff}\right)}$$

$$= \frac{1(1^2 \times 20.7^2 + 4 \times 20.7 \times 15.2^3 + 1^2 \times 20.7 \times 15.2 + 15.2^4)}{12(20.7 + 15.2)} = 799.6\ (mm^4)$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.19 \times 210{,}000 \times 799.6}}{35.9} = 315.16\ (N/mm^2)$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/315.16} = 1.054 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.054 = 0.708$$

Since $\chi_d = 0.708 \neq 0.671$ from previous iteration, more iterations are carried out and the final iteration gives:

$\chi_d = 0.704.$

$b_{e2} = 20.38\ (mm)$

$c_{eff} = 14.95\ (mm)$

$b_{e1} = 18.1\ (mm)$

The neutral axis of the section with effective top flange and lip measured from the centerline of the top flange is:

$$\bar{y} = \frac{\sum_i A_i y_i}{A} = \frac{tb_{e1} \times 0 + \chi_d tb_{e2} \times 0 + \chi_d tc_{eff} \times \frac{c_{eff}}{2} + th_p \frac{h_p}{2} + tb_p h_p + tc_p \left(h_p - \frac{c_p}{2}\right)}{tb_{e1} + \chi_d tb_{e2} + \chi_d tc_{eff} + th_p + tb_p + tc_p} =$$
$$= \frac{0.704 \times 1 \times 14.95 \times \frac{14.95}{2} + 1 \times 149 \frac{149}{2} + 1 \times 46 \times 149 + 1 \times 15.5 \left(149 - \frac{15.5}{2}\right)}{1 \times 18.1 + 0.704 \times 1 \times 20.38 + 0.704 \times 1 \times 14.95 + 1 \times 149 + 1 \times 46 + 1 \times 15.5}$$
$$= \frac{20222.55}{253.47} = 79.78\ (mm)$$

The web is considered an internal (stiffened) element under stress gradient:

$$\sigma_1 = f_{yb} = 350\ (N/mm^2)$$
$$\sigma_2 = -f_{yb} \frac{149 - 79.78}{79.78} = -303.67\ (N/mm^2)$$
$$\psi = \frac{\sigma_2}{\sigma_1} = -\frac{303.67}{350} = -0.867$$
$$k_\sigma = 7.81 - 6.29\psi + 9.78\psi^2 = 7.81 - 6.29 \times (-0.867) + 9.78 \times (-0.867)^2 = 20.62$$
$$\bar{\lambda}_{p,b} = \frac{h_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{149/1}{28.4 \times 0.8194\sqrt{20.62}} = 1.41 > 0.673$$
$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3 + \psi)}{\bar{\lambda}_{p,b}^2} = \frac{1.41 - 0.055(3 - 0.867)}{1.41^2} = 0.65 \leq 1.0$$
$$h_c = \frac{h_p}{(1 - \psi)} = \frac{149}{(1 + 0.867)} = 79.807\ (mm)$$
$$h_t = h_p - h_c = 149 - 79.807 = 69.193\ (mm)$$
$$h_{eff} = \rho \frac{h_p}{(1 - \psi)} = 0.65 \times \frac{149}{(1 + 0.867)} = 51.886\ (mm)$$
$$h_{e1} = 0.4 h_{eff} = 0.4 \times 51.886 = 20.75\ (mm)$$
$$h_{e2} = 0.6 h_{eff} = 0.6 \times 51.886 = 31.13\ (mm)$$

The neutral axis of the section with effective top flange and lip measured from the centerline of the top flange is:

$$\bar{y} = \frac{\sum_i A_i y_i}{A} = 85.363\ (mm)$$
$$W_{eff,c} = 8849.77\ (mm^3)$$
$$W_{eff,t} = 11871.17\ (mm^3)$$
$$M_{c,Rd} = \frac{W_{eff,c} f_{yb}}{\gamma_{M0}} = \frac{8849.77 \times 350}{1.0} = 3097420\ (N - mm)$$

Similar calculation is repeated to determine the effective section modulus about the minor axis subjected to moment $\Delta M_{z,Ed} = N_{Ed} e_{Nz} = 9680\ (N - mm)$ due to the shift of centroidal axis of the effective section under uniform compression as shown previously. Under this moment, the web is in uniform compression and its effective width was found to be 43.344 (mm), and the neutral axis of the cross-section with this effective width and full unreduced flange and lip widths is 21.293 (mm). This neutral axis is closer to the web than the lips, resulting in the tensile stress exceeding the yield stress in the lips and part of the flanges. Therefore, the partially plastic section modulus is calculated with the assumption of stress distribution being bilinear on the tension side and linear on the compression side. The stress ratio in the flanges is also assumed to be -1.0. Example of calculation of partially plastic section modulus can be found in Ex005 and Ex006. For the C section considered in this example, the partially plastic section modulus for bending about minor axis is determined to be:

$W_{eff,z,c} = W_{eff,z,t} = 2484.5\ (mm^4)$

$$M_{cz,Rd} = \frac{W_{eff,z,t} f_{yb}}{\gamma_{M0}} = \frac{2484.5 \times 350}{1.0} = 869575\ (N-mm)$$

2. Lateral-torsional buckling:

Due to the concentrated loading and simply support condition at both ends of the column:

$C_1 = 1.365$, $C_2 = 0.553$, $C_3 = 1.73$

$k_w = 1.0$ and $K_{LTB} = 1.0$

$z_a = 74.5\ (mm)$ as the load is applied on the top flange

$z_g = z_a - z_s = 74.5 - 0 = 74.5\ (mm)$

$z_j = 0.0\ (mm)$ along z-z axis

$L_{cr} = 3000\ (mm)$

$I_t = 90.667\ (mm^4)$

$I_z = 84362.534\ (mm^4)$

$I_w = 392274920\ (mm^6)$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L_{cr}^2} \left\{ \left[\left(\frac{K_{LTB}}{k_w} \right) \frac{I_w}{I_z} + \frac{L_{cr}^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j \right)^2 \right]^{0.5} - \left(C_2 z_g - C_3 z_j \right) \right\}$$

$$= 1082039\ (N-mm)$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y} f_{yb}}{M_{cr}}} = \sqrt{\frac{8847.77 \times 350}{1082039}} = 1.692$$

The applicable buckling curve is b and $\alpha_{LT} = 0.34$

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right] = 0.5[1 + 0.34(1.692 - 0.2) + 1.692^2] = 2.185$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{2.185 + \sqrt{2.185^2 - 1.692^2}} = 0.28 \le 1.0$$

$$M_{b,Rd} = \chi_{LT} W_{eff,y} \frac{f_{yb}}{\gamma_{M1}} = 0.28 \times 8849.77 \frac{350}{1.0} = 868206\ (N-mm)$$

Member Shear Capacity:

$$\bar{\lambda}_w = 0.346\frac{s_w}{t}\sqrt{\frac{f_{yb}}{E}} = 0.346\frac{149}{1}\sqrt{\frac{350}{210000}} = 2.1047$$

$$f_{bv} = \frac{0.67f_{yb}}{\bar{\lambda}_w^2} = \frac{0.67 \times 350}{2.1047^2} = 52.937\ (N/mm^2)$$

$$V_{b,Rd} = \frac{h_w t f_{bv}}{\gamma_{M0}} = \frac{149 \times 1 \times 52.937}{1.0} = 7888\ (N)$$

Combined D/C ratio:

Since $N_{b,Rd} = 15090.6\ (N) < 44255\ (N) = N_{c,Rd}$ and $M_{b,Rd} = 2350508.7\ (N-mm) < 3097420\ (N-mm) = M_{c,Rd}$, the combination ratio is larger for member buckling resistance. Also, by assuming $k_{yy} = k_{zz} = k_{zy} = k_{yz} = 1.0$, the ratio by Equation 6.36 in Eurocode 3 1-3 2006 would provide the largest D/C ratio and govern the design

Eq. 6.36 in EC3 1-3:

$$\frac{D}{C} = \left(\frac{N_{Ed}}{N_{b,Rd}}\right)^{0.8} + \left(\frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{by,Rd}}\right)^{0.8} + \left(\frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{bz,Rd}}\right)^{0.8}$$

$$= \left(\frac{2200}{15090.6}\right)^{0.8} + \left(\frac{585000 + 0}{868206}\right)^{0.8} + \left(\frac{0 + 9680}{869575}\right)^{0.8} = 0.971$$